I-1. Characteristics of Log-Periodic Transmission Line Circuits

R. H. DuHamel and M. E. Armstrong

Hughes Aircraft Company, Fullerton, Calif.

The objectives of this and the companion papers following are to introduce and explain new concepts for the design of transmission line circuits which will theoretically provide frequency-independent performance. All of the circuits are constructed according to log-periodic design principles. As with the corresponding log-periodic antennas, these circuits may be designed to give essentially frequency-independent performance over any desired finite bandwidth. Particularly useful circuits to be described are three-port constant phase difference circuits and four-port hybrids.

An unlimited variety of log-periodic circuits exists. An example of a simple one-port circuit is shown in Fig. 1. It consists of a transmission line of characteristic impedance Z_{01}, shunt loaded at log-periodic intervals by open-circuit transmission lines of characteristic impedance Z_{02}. The design parameters τ and σ are defined in Fig. 1. The circuit may be considered as a cascade of symmetrical sections consisting of transmission lines shunt loaded by open-circuited transmission lines. Ideally, the circuits should extend indefinitely to the right and there should be an infinity of sections between the feed point and the first stub shown. Notice that the circuit is defined such that

$$V_n(f) = V_{n+1}(\tau f) \quad \text{for all } n \tag{1}$$

where V_n is the voltage at the n-th node and n is integral. It may be shown that, under these conditions, the input reflection coefficient is a periodic function of the logarithm of the frequency. Under certain conditions to be discussed, the magnitude of the input reflection coefficient is unity and the phase of the reflection coefficient varies linearly with the logarithm of the

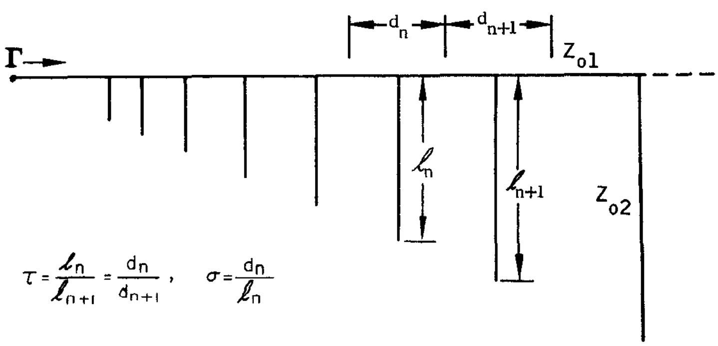

Fig. 1 Simple one-port log-periodic circuit.

frequency. It is this exceedingly simple and amazing property which we make use of in the design of directional couplers, hybrids, and phase difference circuits.

Several two-port and four-port log-periodic circuits are illustrated in Fig. 2. Combinations of the two ports may be used to build constant phase difference circuits. The design procedure for a multiport circuit consists of the following steps:

1) Basic symmetries of the junction are specified such that the normal modes or eigenvectors of the junction are independent of frequency and the eigenvalues allow the elements of the scattering matrix to assume the desired characteristics.

2) The circuit is constructed such that its performance will be periodic with respect to the logarithm of the frequency.

3) If possible, the design parameters of the circuit are chosen such that the phase of the eigenvalues (or the input reflection coefficients of the normal modes) is a linear function of the logarithm of the frequency. A normal mode analysis of a circuit usually reduces to the analysis of a single transmission line which is shunt and/or series loaded in a log-periodic manner.

4) If possible, the design parameters are chosen so that the relative phase of the eigenvalues (or the reflection coefficients for the normal modes) take on certain values. When the proper phases are achieved, certain elements of the scattering matrix for the junction become approximately zero.

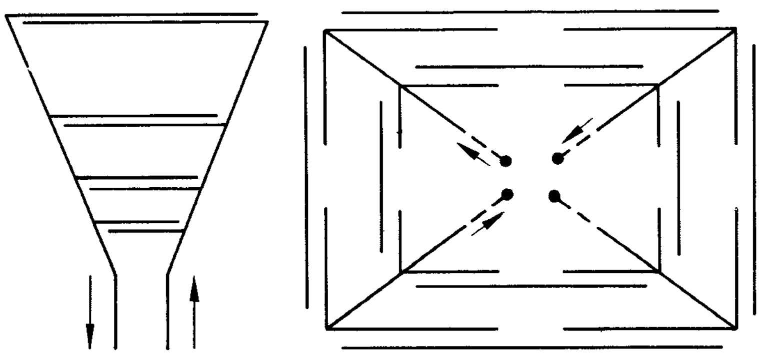

Fig. 2 Log-periodic 2-port coupler and 4-port hybrid networks.

From the above procedures, it is apparent that the successful performance of log-periodic circuits depends critically upon achieving and controlling a linear phase characteristic for a transmission line loaded in a log-periodic manner. The objectives of this paper are to provide a physical insight into the behavior of simple one-port log-periodic transmission line circuits and to present the results of extensive computer investigations.

Returning to Fig. 1, the behavior of the circuit may be explained approximately as follows: An incident wave applied at the input propagates down the structure toward the active region (which is defined as the region where a shunt stub is approximately one-quarter wavelength long). Along the first part of the structure, the loaded transmission line behaves as a slowly varying, nonuniform transmission line. At the active region, the shunt stubs effectively short the wave and reflect it back to the feed point. If complete reflection takes place at the active region, then calculations show that the input reflection coefficient is given ideally and approximately by

$$\Gamma = \exp\left[-j\left(\frac{2\pi \ln f}{|\ln \tau|} + \varphi\right)\right], \tag{2}$$

where f is the frequency and φ is a constant. The reflection coefficient has been normalized to the impedance Z_0, which is given by

$$Z_0 = Z_{01}\left[1 + \frac{Z_{01}}{2\sigma Z_{02}}\right]^{-1/2}. \tag{3}$$

This impedance is simply the limiting form, as $n \to -\infty$, of the image impedance of the symmetrical sections. Note that the phase varies linearly with the logarithm of the frequency, and that the phase changes by 2π when the frequency is changed by the factor τ or a period. Figure 3 shows plots of the ideal phase (solid line) and the actual phase (dashed line) of the reflection coefficient. Note that the actual phase has a log-periodic deviation

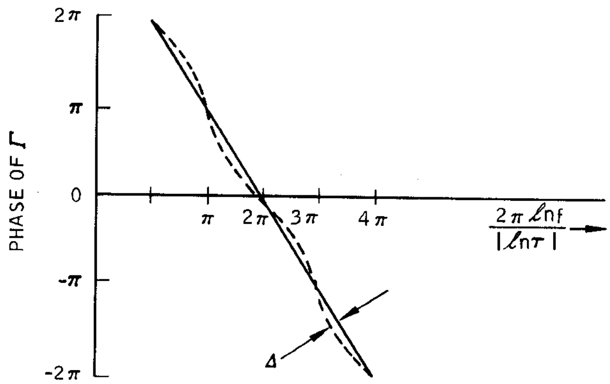

Fig. 3 Stub-loaded line reflection coefficient phase characteristic.

from the desired straight line characteristic. The maximum deviation is denoted by Δ. For given values of σ, Z_{01}/Z_{02}, and Q of the transmission line elements, the above phase characteristic is achieved for τ greater than some minimum value. Among other things, the deviation Δ decreases as σ is increased. Maximum phase deviations of several degrees are ordinary and deviations of only a small fraction of a degree are possible.

If the active region does not effectively short the incident wave for all frequencies in a period, then a portion of the wave will propagate further down the structure for some frequencies and lead to end effect. End effect is evidenced by large phase deviations from the desired linear characteristic and, in some cases, by phase jumps of 360 degrees over a portion of the period. Examples of end effect and the conditions for eliminating end effect are discussed in the paper.

For a structure with a finite number of sections, the performance of the circuit will approach that of the infinite circuit over a finite bandwidth. The lower cutoff frequency is determined when the longest stub is approximately one-quarter wavelength long, and the upper cutoff frequency when the shortest stub is somewhat less than one-quarter wavelength long.

The circuit of Fig. 1 also has an important property which is analogous to the phase rotation principle for log-periodic antennas. That is, if all dimensions of the circuit are scaled gradually from the original dimensions to τ times their starting dimensions, it will be found that the phase curve slides to the right by the amount 2π. This property is useful in the design of phase difference circuits.

Since attempts to derive simple closed form expressions for the reflection coefficient of circuits of the type of Fig. 1 have been unsuccessful, it has been necessary to resort to the use of a high-speed computer for determining their performance. The results of a large number of parameter studies on the circuit of Fig. 1 and similar circuits will be discussed in the paper.